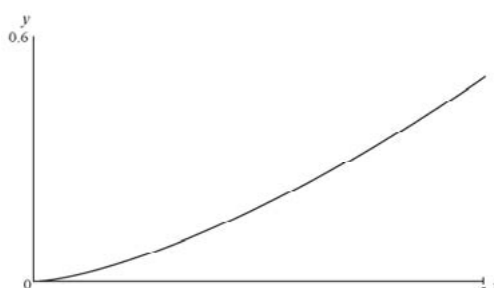


Paper 4E: Further Statistics 2 Mark Scheme

Question	Scheme	Marks	AOs
1(a)	$P(A > 3) = \frac{2}{5}$	B1	1.1b
	$\left(\frac{2}{5}\right)^3 = \frac{8}{125}$	M1 A1	1.1a 1.1b
		(3)	
(b)	$f(y) = \frac{3y^2}{125}$	M1	2.1
	$E(Y) = \int_0^5 \frac{3y^3}{125} dy$ $= \left[\frac{3y^4}{500} \right]_0^5 \quad \left[= \frac{15}{4} \right]$	M1	1.1b
	$\text{Var}(Y) = \int_0^5 \left(\frac{3y^4}{125} \right) dy - \left(\frac{15}{4} \right)^2$	M1	1.1b
	$= 0.9375^*$	A1*cs0	1.1b
		(4)	
(c)	Mode = 5	B1	1.2
	 Or reason based on $\frac{df(y)}{dy} > 0$	B1	2.4
		(2)	
(d)	From a sketch or mode > mean therefore it has negative skew	B1ft	2.4
		(1)	
(e)	$\frac{(2k)^3}{125} - \frac{k^3}{125} = 0.189$	M1	3.1a
	$\frac{7k^3}{125} = 0.189$	A1	1.1b
	$k = 1.5$	A1	1.1b
		(3)	
(13 marks)			

Question 1 notes:**(a)****B1:** $\frac{2}{5}$ o.e. may be implied by a correct answer**M1:** $\left(\text{"their"} \left(\frac{2}{5} \right) \right)^3$ may be implied by a correct answer**A1:** $\frac{8}{125}$ o.e.**(b)****M1:** Realising that firstly need to find pdf $f(y)$ and attempt to differentiate $F(y)$ **M1:** Continuing the argument with an attempt to integrate $y \times \text{"their } f(y)\text{"}$
 $y^n \rightarrow y^{n+1}$ **M1:** Integrating $y^2 \times \text{"their } f(y)\text{"}$ - $[\text{"their } E(Y)\text{"}]^2$ $y^n \rightarrow y^{n+1}$ **A1*:** Complete correct solution no errors**(c)****B1:** 5 only**B1:** Explain their reason by either an accurate sketch or $\frac{df(y)}{dy} > 0$ therefore an increasing function o.e.**(d)****B1ft:** Explaining the reason for their answer. Follow through their part(b) or mean from(d) and mode from(c). A correct sketch of "their $f(y)$ " – may be seen anywhere in question or ft their mean and mode plus a correct conclusion**NB:** Watch for gaming. A student who writes both negative skew with a reason and positive skew with a reason. Please send these to your Team Leader**(e)****M1:** Attempting to translate the problem into an equation using $2k$ and k . Allow if the brackets are missing e.g. $\frac{2k^3}{125} - \frac{k^3}{125}$. No need for the 0.189**A1:** A correct equation in any form**A1:** A correct answer only

Question	Scheme	Marks	AOs
2(a)	$H_0 : \rho = 0, H_1 : \rho > 0$	B1	2.5
	Critical value at 1% level is 0.8929	B1	1.1b
	$r_s < 0.8929$ so not significant evidence to reject H_0	M1	2.1
	The researcher's claim is not correct (at 1% level) or insufficient evidence for researcher's claim or there is insufficient evidence that water gets deeper further from inner bank or no (positive) correlation between depth of water and distance from inner bank	A1ft	2.2b
		(4)	
(b)(i)	The ranks will remain the same therefore there will be no change to the spearman's rank correlation coefficient	B1	2.4
(ii)	Spearman's rank correlation coefficient will increase since	B1	2.2a
	The ranks are the same for both distance and depth therefore $d = 0$ however, n has increased or the new position follows the pattern that large b is associated with large s and so r_s will increase	B1	2.4
		(3)	
(c)	The mean of the tied ranks is given to each...	B1	2.4
	... then use PMCC	B1	2.4
		(2)	
(9 marks)			
Notes:			
(a)			
B1: Both hypotheses correct written using the notation ρ			
B1: awrt 0.893			
M1: Drawing a correct inference using their answer to part(a) and their CV			
A1ft: Drawing a correct inference in context using their answer to part(a) and their CV			
(b)(i)			
B1: Stating no change and an explanation including ranks remain unchanged o.e. and no change o.e.			
(b)(ii)			
B1: Interpreted the outcome of adding a point as increased oe			
B1: Explaining why. Need to mention the ranks are the same for both oe and n has increased oe			
(c)			
B1: Explaining that the mean of the values for the tied ranks is given to both values			
B1: Explaining that the PMCC must be used			

Question	Scheme	Marks	AOs
3(a)	95% CI for μ uses t value of 2.064	B1	3.3
	$\frac{\hat{\sigma}}{\sqrt{25}} \times "2.064" = \frac{1}{2}(2.232 - 1.128)$ or $\frac{1}{2}(2.232 + 1.128) + "2.064" \times \frac{\hat{\sigma}}{\sqrt{25}} = 2.232$ (oe)	M1	2.1
	$\hat{\sigma} = \frac{2.76}{"2.064"} \text{ or } 1.3372\dots$	M1	1.1b
	$\hat{\sigma}^2 = 1.788\dots [=1.79 \text{ (3sf)}]$ *	A1*cso	1.1b
		(4)	
(b)	$12.401, < \frac{24 \times 1.79}{\sigma^2} < 39.364$	B1	1.1b
		M1	1.1a
	<u>1.09</u> < σ^2 < <u>3.46</u>	A1	1.1b
		(3)	
(7 marks)			
Notes:			
(a)			
B1: Realising that the t -distribution must be used as a model and finding the correct value awrt 2.06			
M1: Using the correct formula with a t -value, $\frac{\hat{\sigma}}{\sqrt{25}} \times "t \text{ value}" = \frac{1}{2}(2.232 - 1.128)$ or $\frac{1}{2}(2.232 + 1.128) + "t \text{ value}" \times \frac{\hat{\sigma}}{\sqrt{25}} = 2.232$ or $\frac{1}{2}(2.232 + 1.128) - "t \text{ value}" \times \frac{\hat{\sigma}}{\sqrt{25}} = 1.128$			
M1: Rearranging one of these formula accurately to find a value of $\hat{\sigma}$			
A1cso*: A correct solution only using awrt 1.79			
(b)			
B1: awrt 12.4 or 39.4 May be implied by a correct confidence interval			
M1: $\frac{24 \times 1.79}{\sigma^2}$ May be implied by a correct confidence interval			
A1: awrt 1.09 and awrt 3.46			

Question	Scheme	Marks	AOs
4(a)	$H_0: \sigma_G^2 = \sigma_B^2, H_1: \sigma_G^2 \neq \sigma_B^2,$	B1	2.5
	$s_B^2 = \frac{1}{6}(56130 - 7 \times 88.9^2) = \frac{807.53}{6} = 134.6$	M1 A1	2.1 1.1b
	$s_G^2 = \frac{1}{7}(55746 - 8 \times 83.1^2) = \frac{501.12}{7} = 71.58$	A1	1.1b
	$\frac{s_B^2}{s_G^2} = 1.880...$	M1	3.4
	Critical value $F_{6,7} = 3.87$	B1	1.1b
	Not significant, variances can be treated as the same	A1 ft	2.2b
		(7)	
(b)	$H_0: \mu_B = \mu_G, H_1: \mu_B > \mu_G$	B1	2.5
	Pooled estimate of variance $s^2 = \frac{6 \times 134.6 + 7 \times 71.58}{13} = 100.6653...$	M1	3.1b
	Test statistic $t = \frac{88.9 - 83.1}{s \sqrt{\frac{1}{7} + \frac{1}{8}}} = \text{awrt } 1.12$	M1 A1	1.1b 1.1b
	Critical value $t_{13}(5\%) = 1.771$	B1	1.1b
	Insufficient evidence to support mother's claim	A1 ft	2.2b
		(6)	
(13 marks)			
Notes:			
(a)			
B1: Both hypotheses correct using the notation σ^2 . Allow σ rather than σ^2			
M1: Using a correct Method for either s_B^2 or s_G^2 May be implied by a correct value			
A1: awrt 135			
A1: awrt 71.6			
M1: Using the F-distribution as the model e.g. $\frac{s_B^2}{s_G^2}$			
B1: awrt 3.87			
A1ft: Drawing a correct inference following through their CV and value for $\frac{s_B^2}{s_G^2}$			
(b)			
B1: Both hypotheses correct using the notation μ			
M1: For realising the need to find the pooled estimate for the test require from a correct interpretation of the question			
M1: Correct method for test statistic $t = \frac{88.9 - 83.1}{\text{"their } s" \sqrt{\frac{1}{7} + \frac{1}{8}}}$ May be implied by a correct awrt 1.12			
A1: awrt 1.12			
B1: awrt 1.77			
A1ft: Drawing a correct inference following through their CV and test statistic			

Question	Scheme	Marks	AOs
5(a)	Let $X = L - 4S$ then $E(X) = 19.6 - 4 \times 4.8$	M1	2.3
	$= 0.4$	A1	1.1b
	$\text{Var}(X) = \text{Var}(L) + 4^2 \text{Var}(S) = 0.6^2 + 16 \times 0.3^2$	M1	2.1
	$= 1.8$	A1	1.1b
	$P(X > 0) = [P(Z > \frac{0-0.4}{\sqrt{1.8}} = -0.298... \dots)]$	M1	2.1
	$= 0.617202... \quad \text{awrt } \underline{0.617}$	A1	1.1b
		(6)	
(b)	$T = S_1 + S_2 + S_3 + S_4$ (May be implied by 0.36)	M1	3.3
	$T \sim N(19.2, 0.36) \quad E(T) = 19.2$	B1	1.1b
	$\text{Var}(T) = 0.36 \quad \text{or} \quad 0.6^2$	A1	1.1b
		(3)	
(c)	Let $Y = L - T \quad E(Y) = E(L) - E(T) = [0.4]$	M1	3.3
	$\text{Var}(Y) = \text{Var}(L) + \text{Var}(T) = [0.72]$	M1	1.1b
	Require $P(-0.2 < Y < 0.2)$	M1	3.1a
	$= 0.16708... \quad \text{awrt } \underline{0.167}$	A1	1.1b
		(4)	
(13 marks)			
Notes:			
(a)			
M1: Selecting and using an appropriate model i.e. $\pm(L - 4S)$. May be implied by 0.4			
A1: 0.4 oe			
M1: For realising the need to use $\text{Var}(L) + 4^2 \text{Var}(S)$. Allow use of 0.6 for $\text{Var}(L)$ instead of 0.6^2 and/or 0.3 for $\text{Var}(S)$ instead of 0.3^2 may be implied by 1.8			
A1: 1.8 only			
M1: For realising $P(X > 0)$ is required and an attempt to find it e.g. $\frac{0-0.4}{\sqrt{\text{"their Var}(X)}}$ but do not allow a negative $\text{Var}(X)$			
A1: awrt 0.617			
(b)			
M1: Selecting and using an appropriate model ie $S_1 + S_2 + S_3 + S_4$: may be implied by 0.36			
B1: 19.2 only			
A1: 0.36			
(c)			
M1: Setting up and using the model $Y = L - T$. May be implied by $E(Y) = E(L) - E(T)$			
M1: Using $\text{Var}(Y) = \text{Var}(L) + \text{Var}(T)$			
M1: Dealing with the modulus and realising they need to find $P(-0.2 < Y < 0.2)$			
A1: awrt 0.167			

Question	Scheme	Marks	AOs																																												
6(a)	$\left[b = \frac{S_{xm}}{S_{xx}} = -0.0277576 \right]$	M1	3.3																																												
	$[a = \bar{m} - b\bar{x} = 1.278 + 0.0277576 \times 8.5 = 1.5139]$																																														
	$m = 1.5139 - 0.02775 \dots x$	A1	1.1b																																												
		(2)																																													
(b)	$RSS = 0.12756 - \frac{(-2.29)^2}{82.5}$	M1	1.1b																																												
	$= 0.06399^*$	A1*	1.1b																																												
		(2)																																													
(c)	<table><tr><td>x</td><td>m</td><td>$m = a + bx$</td><td>ε</td></tr><tr><td>4</td><td>1.50</td><td>1.4029</td><td>+0.0971</td></tr><tr><td>5</td><td>1.20</td><td>1.3752</td><td>-0.1752</td></tr><tr><td>6</td><td>1.40</td><td>1.3474</td><td>+0.0526</td></tr><tr><td>7</td><td>1.40</td><td>1.3196</td><td>+0.0804</td></tr><tr><td>8</td><td>1.23</td><td>1.2919</td><td>-0.0619</td></tr><tr><td>9</td><td>1.30</td><td>1.2641</td><td>+0.0359</td></tr><tr><td>10</td><td>1.20</td><td>1.2364</td><td>-0.0364</td></tr><tr><td>11</td><td>1.15</td><td>1.2086</td><td>-0.0586</td></tr><tr><td>12</td><td>1.25</td><td>1.1808</td><td>+0.0692</td></tr><tr><td>13</td><td>1.15</td><td>1.1531</td><td>-0.0031</td></tr></table>	x	m	$m = a + bx$	ε	4	1.50	1.4029	+0.0971	5	1.20	1.3752	-0.1752	6	1.40	1.3474	+0.0526	7	1.40	1.3196	+0.0804	8	1.23	1.2919	-0.0619	9	1.30	1.2641	+0.0359	10	1.20	1.2364	-0.0364	11	1.15	1.2086	-0.0586	12	1.25	1.1808	+0.0692	13	1.15	1.1531	-0.0031	M1 A1	3.4 1.1b
	x	m	$m = a + bx$	ε																																											
	4	1.50	1.4029	+0.0971																																											
	5	1.20	1.3752	-0.1752																																											
	6	1.40	1.3474	+0.0526																																											
	7	1.40	1.3196	+0.0804																																											
	8	1.23	1.2919	-0.0619																																											
	9	1.30	1.2641	+0.0359																																											
	10	1.20	1.2364	-0.0364																																											
	11	1.15	1.2086	-0.0586																																											
	12	1.25	1.1808	+0.0692																																											
	13	1.15	1.1531	-0.0031																																											
		(2)																																													
(d)	The point (5, 1.2) is an outlier	B1ft	2.2b																																												
		(1)																																													
(e)(i)	It is a valid piece of data so should be used or It does not follow the pattern according to the residuals so may contain an error making the result invalid so should be removed	B1	2.4																																												
(ii)	$a = \bar{m} - b\bar{x} = 1.28667 + 0.03765 \times 8.88889 = 1.6213$	M1	3.3																																												
	$m = 1.6213 - 0.03765x$	A1	1.1b																																												
(iii)	$m = 1.6213 - 0.03765 \times 15$																																														
	$= 1.056$ or awrt 1.06	B1ft	3.4																																												
(iv)	The model is only reliable if the values are limited to those in the given range so probably not reliable	B1	3.5b																																												
		(5)																																													
(12 marks)																																															

Question 6 notes:	
6(a)	
M1:	Realising the need to use $b = \frac{S_{xm}}{S_{xx}}$ and $a = \bar{m} - b\bar{x}$
A1:	$m = \text{awrt } 1.51) - (\text{awrt } 0.0278)x$. Award M1A1 for correct equation
(b)	
M1:	Using $S_{mm} - \frac{(S_{xm})^2}{S_{xx}}$
A1*:	awrt 0.064
(c)	
M1:	Using the model in part (a) i.e. $m = ("1.5139" - "0.02775"x)$ implied by a correct value
A1:	All correct. Award M1A1 for a list of correct residuals
(d)	
B1:	Inferring from the residuals that the outlier is (5, 1.2) ft their residuals.
(e)(i)	
B1:	Explaining why the outlier should be removed or not.
(ii)	
M1:	Removing the outlier and refining the model by finding a new regression line.
A1:	$m = (\text{awrt } 1.62) - (\text{awrt } 0.0377)x$
(iii)	
B1ft:	using their model in e(i) with $x = 15$. awrt 1.06 or ft their e(ii)
(iv)	
B1:	Realising the limitations of the model by stating it is <u>not reliable</u> and giving the reason why i.e. extrapolation/out of range o.e.

Question	Scheme	Marks	AOs
7(a)	$S_{xx} = \sum (10s)^2 - \frac{(\sum 10s)^2}{10}$	M1	2.1
	$2658.9 = 100 \sum (s)^2 - \frac{100(\sum s)^2}{10}$	M1	1.1b
	$2658.9 = 100 S_{ss}$		
	$S_{ss} = 26.589 *$	A1*cso	1.1b
		(3)	
(b)	$64 = \sum_{i=1}^{10} 10(d_i - 9)$	M1	3.1a
	$64 = 10 \sum_{i=1}^{10} d_i - 900$		
	$\sum_{i=1}^{10} d_i = 96.4$	A1	1.1b
	$S_{dd} = 1081.74 - \frac{("96.4")^2}{10}$	M1	1.1b
	$= 152.444$		
	$r = 0.935$	A1ft	1.1b
		(4)	
(c)	Linear correlation is significant but scatter diagram suggests a non-linear relationship between the level of serum magnesium, and the level of the disease protein	B1	3.5a
		(1)	
(8 marks)			
Notes:			
(a)			
M1: Attempting to use $S_{xx} = \sum x^2 - \frac{(\sum x)^2}{10}$ with $x = 10s$			
M1: Substituting in 2658.9 and dealing with the 10 correctly			
A1*: cso A complete solution with no errors leading to 26.589 only			
(b)			
M1: Realising that either $64 = \sum_{i=1}^{10} 10(d_i - 9)$ or $64 = 10 \sum_{i=1}^{10} d_i - 900$ o.e. must be used. May be implied by seeing 96.4			
A1: 96.4 only			
M1: Attempting to use $S_{dd} = \sum d^2 - \frac{(\sum d)^2}{10}$ may be implied by 0.935			
A1ft: awrt 0.935 ft “their 96.4”			
(c)			
B1: A correct comment comparing their value of r and the scatter diagram in context			

